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THE GRAND UNIFIED THEORY OF CLASSICAL QUANTUM MECHANICS

Randell L. Mills*

1. INTRODUCTION

A theory of classical quantum mechanics (CQM), derived from first principles,[1] successfully applies physical laws on all scales. The classical wave equation is solved with the constraint that a bound electron cannot radiate energy. The mathematical formulation for zero radiation based on Maxwell's equations follows from a derivation by Haus.[2] The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. CQM gives closed form solutions for the atom, including the stability of the n=1 state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, **r x p**, can be applied directly to the wave function, called an orbitsphere (a current density function), that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling are derived in closed form equations based on Maxwell's equations. The calculations agree with experimental observations.

For or any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. By applying this condition to electromagnetic and gravitational fields at

* Randell L Mills, President, BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ 08512, Phone: 609-490-1090, e-mail: rmills@blacklightpower.com; www.blacklightpower.com

particle production, the Schwarzschild metric (SM) is derived from the classical wave equation which modifies general relativity to include conservation of spacetime in addition to momentum and matter/energy. The result gives a natural relationship between Maxwell's equations, special relativity, and general relativity. It gives gravitation from the atom to the cosmos. The Universe is time harmonically oscillatory in matter energy and spacetime expansion and contraction with a minimum radius that is the gravitational radius. In closed form equations with fundamental constants only, CQM gives the deflection of light by stars, the precession of the perihelion of Mercury, the particle masses, the Hubble constant, the age of the Universe, the observed acceleration of the expansion, the power of the Universe, the power spectrum of the Universe, the microwave background temperature, the uniformity of the microwave background radiation, the microkelvin spatial variation of the microwave background radiation, the observed violation of the GZK cutoff, the mass density, the large scale structure of the Universe, and the identity of dark matter which matches the criteria for the structure of galaxies. In a special case wherein the gravitational potential energy density of a blackhole equals that of the Plank mass, matter converts to energy and spacetime expands with the release of a gamma-ray burst. The singularity in the SM is eliminated.

2. COSMOLOGICAL THEORY BASED ON MAXWELL'S EQUATIONS

Maxwell's equations and special relativity are based on the law of propagation of a electromagnetic wave front in the form

$$1/c^2(\delta\omega/\delta t)^2 - \left[(\delta\omega/\delta x)^2 + (\delta\omega/\delta y)^2 + (\delta\omega/\delta\omega)^2\right] = 0 \tag{1}$$

For any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. Thus, the equation $1/c^2(\delta\omega/\delta t)^2 - (grad\omega)^2 = 0$ acquires a general character; it is more general than Maxwell's equations from which Maxwell originally derived it.

A discovery of the present work is that the classical wave equation governs: (1) the motion of bound electrons, (2) the propagation of any form of energy, (3) measurements between inertial frames of reference such as time, mass, momentum, and length (Minkowski tensor), (4) fundamental particle production and the conversion of matter to energy, (5) a relativistic correction of spacetime due to particle production or annihilation (Schwarzschild metric), (6) the expansion and contraction of the Universe, (7) the basis of the relationship between Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and special, and general relativity.

The relationship between the time interval between ticks t of a clock in motion with velocity v relative to an observer and the time interval t_0 between ticks on a clock at rest relative to an observer[3] is

$$(ct)^2 = (ct_0)^2 + (vt)^2. \tag{2}$$

Thus, the time dilation relationship based on the constant maximum speed of light c in any inertial frame is $t = t_0\Big/\sqrt{1-\left(v^2/c^2\right)}$. The metric $g_{\mu\nu}$ for Euclidean space is the

Minkowski tensor $\eta_{\mu\nu}$. In this case, the separation of proper time between two events x^μ and $x^\mu + dx^\mu$ is $d\tau^2 = -\eta_{\mu\nu}dx^\mu dx^\nu$.

3. THE EQUIVALENCE OF THE GRAVITATIONAL MASS AND THE INERTIAL MASS

The equivalence of the gravitational mass and the inertial mass m_g/m_i = universal constant which is predicted by Newton's law of mechanics and gravitation is experimentally confirmed to less $1 X 10^{-11}$.[4] In physics, the discovery of a universal constant often leads to the development of an entirely new theory. From the universal constancy of the velocity of light c, the special theory of relativity was derived; and from Planck's constant h, the quantum theory was deduced. Therefore, the universal constant m_g/m_i should be the key to the gravitational problem. The energy equation of Newtonian gravitation is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \text{constant} \tag{3}$$

Since h, the angular momentum per unit mass, is $h = L/m = |\mathbf{r} \times \mathbf{v}| = r_0 v_0 \sin\phi$, the eccentricity e may be written as

$$e = [1 + \left(v_0^2 - \frac{2GM}{r_0}\right)\frac{r_0^2 v_0^2 \sin^2\phi}{G^2M^2}]^{1/2}, \tag{4}$$

where m is the inertial mass of a particle, v_0 is the speed of the particle, r_0 is the distance of the particle from a massive object, ϕ is the angle between the direction of motion of the particle and the radius vector from the object, and M is the total mass of the object (including a particle). The eccentricity e given by Newton's differential equations of motion in the case of the central field permits the classification of the orbits according to the total energy E [5] (column 1) and the orbital velocity squared, v_0^2, relative to the gravitational velocity squared, $2GM/r_0$ [5] (column 2):

$E < 0$	$v_0^2 < 2GM/r_0$	$e < 1$	ellipse
$E < 0$	$v_0^2 < 2GM/r_0$	$e = 0$	circle (special case of ellipse)
$E = 0$	$v_0^2 = 2GM/r_0$	$e = 1$	parabolic orbit
$E > 0$	$v_0^2 > 2GM/r_0$	$e > 1$	hyperbolic orbit

4. CONTINUITY CONDITIONS FOR THE PRODUCTION OF A PARTICLE FROM A PHOTON TRAVELING AT LIGHT SPEED

A photon traveling at the speed of light gives rise to a particle with an initial radius equal to its Compton wavelength bar.

$$r = \lambda_C = \frac{\hbar}{m_0 c} = r_\alpha^*, \tag{5}$$

The particle must have an orbital velocity equal to Newtonian gravitational escape velocity v_g of the antiparticle.

$$v_g = \sqrt{\frac{2Gm}{r}} = \sqrt{\frac{2Gm_0}{\lambda_C}}. \tag{6}$$

The eccentricity is one. The orbital energy is zero. The particle production trajectory is a parabola relative to the center of mass of the antiparticle.

4.1 A Gravitational Field as a Front Equivalent to Light Wave Front

The particle with a finite gravitational mass gives rise to a gravitational field that travels out as a front equivalent to a light wave front. The form of the outgoing gravitational field front traveling at the speed of light is $f(t - r/c)$ and $d\tau^2$ is given by

$$d\tau^2 = f(r)dt^2 - \frac{1}{c^2}\left[f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{7}$$

The speed of light as a constant maximum as well as phase matching and continuity conditions of the electromagnetic and gravitational waves require the following form of the squared displacements:

$$(c\tau)^2 + (v_g t)^2 = (ct)^2, \tag{8}$$

$$f(r) = \left(1 - \left(\frac{v_g}{c}\right)^2\right). \tag{9}$$

In order that the wave front velocity does not exceed c in any frame, spacetime must undergo time dilation and length contraction due to the particle production event. *The derivation and result of spacetime time dilation is analogous to the derivation and result of special relativistic time dilation* wherein the relative velocity of two inertial frames replaces the gravitational velocity.

The general form of the metric due to the relativistic effect on spacetime due to mass m_0 with v_g given by Eq. (6) is

$$d\tau^2 = \left(1 - \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \tag{10}$$

The gravitational radius, r_g, of each orbitsphere of the particle production event, each of

mass m_0 and the corresponding general form of the metric are respectively

$$r_g = \frac{2Gm_0}{c^2}, \tag{11}$$

$$d\tau^2 = \left(1 - \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \tag{12}$$

The metric $g_{\mu\nu}$ for non-euclidean space due to the relativistic effect on spacetime due to mass m_0 is

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2Gm_0}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{c^2}\left(1 - \frac{2Gm_0}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & \frac{1}{c^2}r^2 & 0 \\ 0 & 0 & 0 & \frac{1}{c^2}r^2 \sin^2\theta \end{pmatrix} \tag{12a}$$

Masses and their effects on spacetime *superimpose*. The separation of proper time between two events x^μ and $x^\mu + dx^\mu$ is

$$d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \tag{13}$$

The *Schwarzschild metric[Eq. (12a)] gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity.*

4.2. Particle Production Continuity Conditions from Maxwell's Equations, and the Schwarzchild Metric

The photon to particle event requires a transition state that is continuous wherein the velocity of a transition state orbitsphere is the speed of light. The radius, r, is the Compton wavelength bar, λ_c, given by Eq. (5). At production, the Planck equation energy, the electric potential energy, and the magnetic energy are equal to m_0c^2.

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the masses of fundamental particles. Substitution of $r = \lambda_C$; $dr = 0$; $d\theta = 0$; $\sin^2\theta = 1$ into the Schwarzschild metric gives

$$d\tau = dt\left(1 - \frac{2Gm_0}{c^2 r_\alpha^*} - \frac{v^2}{c^2}\right)^{\frac{1}{2}}, \tag{14}$$

with $v^2 = c^2$, the relationship between the proper time and the coordinate time is

$$\tau = ti\sqrt{\frac{2GM}{c^2 r_\alpha^*}} = ti\sqrt{\frac{2GM}{c^2 \lambda_c}} = ti\frac{v_g}{c}. \tag{15}$$

When the orbitsphere velocity is the speed of light, continuity conditions based on the constant maximum speed of light given by Maxwell's equations are mass energy = Planck equation energy = electric potential energy = magnetic energy = mass/spacetime metric energy. Therefore, $m_o c^2 = \hbar\omega^* = V = E_{mag} = E_{spacetime}$

$$m_0 c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0 \lambda_C^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0 \lambda_C} = \alpha^{-1}\frac{\pi\mu_0 e^2 \hbar^2}{(2\pi m_0)^2 \lambda_C^3} = \frac{\alpha h}{1\,\mathrm{sec}}\sqrt{\frac{\lambda_C c^2}{2Gm}}. \tag{16}$$

The continuity conditions based on the constant maximum speed of light given by the Schwarzschild metric are:

$$\frac{\text{proper time}}{\text{coordinate time}} = \frac{\text{gravitational wave condition}}{\text{electromagnetic wave condition}} = \frac{\text{gravitational mass phase matching}}{\text{charge/inertial mass phase matching}}, \tag{17}$$

$$\frac{\text{proper time}}{\text{coordinate time}} = i\frac{\sqrt{\frac{2Gm}{c^2 \lambda_C}}}{\alpha} = i\frac{v_g}{\alpha c}. \tag{18}$$

5. MASSES OF FUNDAMENTAL PARTICLES

Each of the Planck equation energy, electric energy, and magnetic energy corresponds to a particle given by the relationship between the proper time and the coordinate time. The electron and down-down-up neutron correspond to the Planck equation energy. The muon and strange-strange-charmed neutron correspond to the electric energy. The tau and bottom-bottom-top neutron correspond to the magnetic energy. The particle must possess the escape velocity v_g relative to the antiparticle where $v_g<c$. According to Newton's law of gravitation, the eccentricity is one and the particle production trajectory is a parabola relative to the center of mass of the antiparticle.

5.1. The Electron-Antielectron Lepton Pair

A clock is defined in terms of a self consistent system of units used to measure the particle mass. The proper time of the particle is equated with the coordinate time according to the Schwarzschild metric corresponding to light speed. The special relativistic condition corresponding to the Planck energy gives the mass of the electron.

$$2\pi\frac{\hbar}{mc^2} = sec\sqrt{\frac{2Gm^2}{c\alpha^2\hbar}}, \tag{19}$$

$$m_e = \left(\frac{h\alpha}{\sec c^2}\right)^{\frac{1}{2}}\left(\frac{c\hbar}{2G}\right)^{\frac{1}{4}} = 9.1097\ X\ 10^{-31}\ kg\ , \tag{20}$$

$$m_e = 9.1097\ X\ 10^{-31}\ kg - 18\ eV/c^2\,(\nu_e) = 9.1094\ X\ 10^{-31}\ kg\ , \tag{21}$$

$$m_{e\,experimental} = 9.1095\ X\ 10^{-31}\ kg\ . \tag{22}$$

5.2. Down-Down-Up Neutron (DDU)

The corresponding equation for production of the neutron is

$$2\pi\frac{2\pi\hbar}{\frac{m_N}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]c^2} = \sec\sqrt{\frac{2G\left[\frac{m_N}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]\right]^2}{3c(2\pi)^2\hbar}}\ , \tag{23}$$

$$m_{N\,calculated} = (3)(2\pi)\left(\frac{1}{1-\alpha}\right)\left(\frac{2\pi h}{\sec c^2}\right)^{\frac{1}{2}}\left(\frac{2\pi(3)ch}{2G}\right)^{\frac{1}{4}} = 1.6744X10^{-27}\ kg\ , \tag{24}$$

$$m_{N\,experimental} = 1.6749X10^{-27}\ kg\ . \tag{25}$$

6. GRAVITATIONAL POTENTIAL ENERGY

The gravitational radius, α_G or r_G, of an orbitsphere of mass m_0 is defined as

$$\alpha_G = r_G = \frac{Gm_0}{c^2}\ , \tag{26}$$

when the $r_G = r_\alpha^* = \lambda_C$, the gravitational potential energy equals m_0c^2

$$r_G = \frac{Gm_0}{c^2} = \lambda_c = \frac{\hbar}{m_0c}\ , \tag{27}$$

$$E_{grav} = \frac{Gm_0^2}{r} = \frac{Gm_0^2}{\lambda_C} = \frac{Gm_0^2}{r_\alpha^*} = \hbar\omega^* = m_0c^2\ . \tag{28}$$

The mass m_0 is the Planck mass, m_u,

$$m_u = m_0 = \sqrt{\frac{\hbar c}{G}}\ , \tag{29}$$

the corresponding gravitational velocity, v_G, is defined as

$$v_G = \sqrt{\frac{Gm_0}{r}} = \sqrt{\frac{Gm_0}{\lambda_C}} = \sqrt{\frac{Gm_u}{\lambda_C}}\,. \tag{30}$$

6.1. Relationship of the Equivalent Planck Mass Particle Production Energies

For the Plank mass particle, the relationships corresponding to Eq. (16) are: (mass energy = Planck equation energy = electric potential energy = magnetic energy = gravitational potential energy $\doteq$ mass/spacetime metric energy)

$$m_0c^2 = \hbar\omega^* = V = E_{mag} = E_{grav} = E_{spacetime}, \tag{31}$$

$$m_0c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0\lambda_C^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0\lambda_C} = \alpha^{-1}\frac{\pi\mu_0 e^2\hbar^2}{(2\pi m_0)^2\lambda_C^3} = \alpha^{-1}\frac{\mu_0 e^2 c^2}{2h}\sqrt{\frac{Gm_0}{\lambda_C}}\sqrt{\frac{\hbar c}{G}} = \frac{\alpha h}{1\,\text{sec}}\sqrt{\frac{\lambda_C c^2}{2Gm}} \tag{32}$$

These equivalent energies give the particle masses in terms of the gravitational velocity, v_G, and the Planck mass, m_u

$$m_0 = \alpha^{-1}\frac{\mu_0 e^2 c}{2h}\frac{\sqrt{\frac{Gm_0}{\lambda_C}}}{c}m_u = \alpha^{-1}\frac{\mu_0 e^2 c}{2h}\sqrt{\frac{Gm_0}{c^2\lambda_C}}m_u = \alpha^{-1}\frac{\mu_0 e^2 c}{2h}\frac{v_G}{c}m_u = \frac{v_G}{c}m_u\,. \tag{33}$$

6.2. Planck Mass Particles

A pair of particles each of the Planck mass corresponding to the gravitational potential energy is not observed since the velocity of each transition state orbitsphere is the gravitational velocity, v_G that in this case is the speed of light; whereas, the Newtonian gravitational escape velocity v_g is $\sqrt{2}$ the speed of light. In this case, an electromagnetic wave of mass energy equivalent to the Planck mass travels in a circular orbit about the center of mass of another electromagnetic wave of mass energy equivalent to the Planck mass wherein the eccentricity is equal to zero and the escape velocity can never be reached. The Planck mass is a "measuring stick." The extraordinarily high Planck mass ($\sqrt{\hbar c/G} = 2.18\,X\,10^{-8}\,kg$) is the unobtainable mass bound imposed by the angular momentum and speed of the photon relative to the gravitational constant. It is analogous to the unattainable bound of the speed of light for a particle possessing finite rest mass imposed by the Minkowski tensor.

6.3. Astrophysical Implications of Planck Mass Particles

The limiting speed of light eliminates the singularity problem of Einstein's equation that arises as the radius of a blackhole equals the Schwarzschild radius. General relativity with the singularity eliminated resolves the paradox of the infinite propagation velocity required for the gravitational force in order to explain why the angular momentum of objects orbiting a gravitating body does not increase due to the finite propagation delay of the gravitational force according to special relativity.[6] When the gravitational potential energy density of a massive body such as a blackhole equals that of a particle having the Planck mass, the matter may transition to photons of the Planck mass. Even light from a

blackhole will escape when the decay rate of the trapped matter with the concomitant spacetime expansion is greater than the effects of gravity which oppose this expansion. Gamma-ray bursts are the most energetic phenomenon known that can release an explosion of gamma rays packing 100 times more energy than a supernova explosion.[7] The annihilation of a blackhole may be the source of *γ-ray bursts*. The source may be due to conversion of matter to photons of the Planck mass/energy which may also give rise to cosmic rays which are the most energetic particles known, and their origin is also a mystery.[8] According to the GZK cutoff, the cosmic spectrum cannot extend beyond 5 X $10^{19} eV$, but AGASA, the world's largest air shower array, has shown that the spectrum is extending beyond $10^{20} eV$ without any clear sign of cutoff.[9] Photons each of the Planck mass may be the source of these inexplicably energetic cosmic rays.

7. RELATIONSHIP OF MATTER TO ENERGY AND SPACETIME EXPANSION

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime. The limiting velocity c results in the contraction of spacetime due to particle production, which is given by $2\pi r_g$ where r_g is the gravitational radius of the particle. This has implications for the expansion of spacetime when matter converts to energy. Q the mass/energy to expansion/contraction quotient of spacetime is given by the ratio of the mass of a particle at production divided by T the period of production.

$$Q = \frac{m_0}{T} = \frac{m_0}{\frac{2\pi r_g}{c}} = \frac{m_0}{\frac{2\pi \frac{2Gm_0}{c^2}}{c}} = \frac{c^3}{4\pi G} = 3.22\ X\ 10^{34} \frac{kg}{sec}. \tag{34}$$

The gravitational equations with the equivalence of the particle production energies [Eq. (16)] permit the conservation of mass/energy ($E=mc^2$) and spacetime ($c^3/4\pi G=3.22$ X 10^{34} kg/sec). With the conversion of 3.22 X 10^{34} kg of matter to energy, spacetime expands by 1 sec. The photon has inertial mass and angular momentum, but due to Maxwell's equations and the implicit special relativity it does not have a gravitational mass.

7.1. Cosmological Consequences

The Universe is closed (it is *finite but with no boundary*). It is a 3-sphere Universe-Riemannian three dimensional hyperspace plus time of constant positive curvature at each r-sphere. The *Universe is oscillatory in matter/energy and spacetime* with a finite minimum radius, the gravitational radius. Spacetime expands as mass is released as energy which provides the basis of the atomic, thermodynamic, and cosmological arrows of time. Different regions of space are isothermal even though they are separated by greater distances than that over which light could travel during the time of the expansion of the Universe.[10] Presently, stars and large scale structures exist which are older than the elapsed time of the present expansion as stellar, galaxy, and supercluster evolution occurred during the contraction phase.[11-17] The maximum power radiated by the Universe which occurs at the beginning of the expansion phase is $P_U=c^5/4\pi G$ = $2.89\ X\ 10^{51} W$.

Observations beyond the beginning of the expansion phase are not possible since the Universe is entirely matter filled.

7.2. The Period of Oscillation of the Universe Based on Closed Propagation of Light

Mass/energy is conserved during harmonic expansion and contraction. The gravitational potential energy E_{grav} given by Eq. (28) with $m_0 = m_U$ is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G. The gravitational velocity v_G [Eq. (30) with $r = r_G$ and $m_0 = m_U$] is the speed of light in a circular orbit wherein the eccentricity is equal to zero and the escape velocity from the Universe can never be reached. The period of the oscillation of the Universe and the period for light to transverse the Universe corresponding to the gravitational radius r_G must be equal. The harmonic oscillation period, T, is

$$T = \frac{2\pi r_G}{c} = \frac{2\pi G m_U}{c^3} = \frac{2\pi G\left(2 X 10^{54}\ kg\right)}{c^3} = 3.10 X 10^{19}\ sec = 9.83 X 10^{11}\ \text{years}, \tag{35}$$

where the mass of the Universe, m_U, is approximately $2 X 10^{54} kg$. (The initial mass of the Universe of $2 X 10^{54} kg$ is based on internal consistency with the size, age, Hubble constant, temperature, density of matter, and power spectrum.) Thus, the observed Universe will expand as mass is released as photons for $4.92 X 10^{11}$ *years*. At this point in its world line, the Universe will obtain its maximum size and begin to contract.

8. THE DIFFERENTIAL EQUATION OF THE RADIUS OF THE UNIVERSE

Based on conservation of mass/energy ($E=mc^2$) and spacetime ($c^3/4\pi G = 3.22 X 10^{34}$ kg/sec). The Universe behaves as a simple harmonic oscillator having a restoring force, **F**, which is proportional to the radius. The proportionality constant, k, is given in terms of the potential energy, E, gained as the radius decreases from the maximum expansion to the minimum contraction.

$$\frac{E}{\aleph^2} = k. \tag{36}$$

Since the gravitational potential energy E_{grav} is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G

$$F = -k\aleph = -\frac{m_U c^2}{r_G^2}\aleph = -\frac{m_U c^2}{\left(\frac{G m_U}{c^2}\right)^2}\aleph. \tag{37}$$

And, the differential equation of the radius of the Universe, $\aleph$ is

$$m_U \ddot{\aleph} + \frac{m_U c^2}{r_G^2}\aleph = m_U \ddot{\aleph} + \frac{m_U c^2}{\left(\frac{G m_U}{c^2}\right)^2}\aleph = 0. \tag{38}$$

The *maximum radius of the Universe*, the amplitude, r_0, of the time harmonic variation in the radius of the Universe, is given by the quotient of the total mass of the Universe and Q, the mass/energy to expansion/contraction quotient.

$$r_0 = \frac{m_U}{Q} = \frac{m_U}{\frac{c^3}{4\pi G}} = \frac{2\,X\,10^{54}\,kg}{\frac{c^3}{4\pi G}} = 1.97\,X\,10^{12}\ light\ years. \tag{39}$$

The *minimum radius* which corresponds to the gravitational radius r_g, given by Eq. (11) with $m_0=m_U$ is 3.12 X 10^{11} *light years.* When the radius of the Universe is the gravitational radius, r_g, the proper time is equal to the coordinate time by Eq. (15) and the gravitational escape velocity v_g of the Universe is the speed of light. The radius of the Universe as a function of time is

$$\aleph = \left(r_g + \frac{cm_U}{Q}\right) - \frac{cm_U}{Q}\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right) = \left(\frac{2Gm_U}{c^2} + \frac{cm_U}{\frac{c^3}{4\pi G}}\right) - \frac{cm_U}{\frac{c^3}{4\pi G}}\cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right). \tag{40}$$

The expansion/contraction rate, $\dot{\aleph}$, is given by time derivative of Eq. (40)

$$\dot{\aleph} = 4\pi c\,X\,10^{-3}\sin\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\frac{km}{sec}. \tag{41}$$

9. THE HUBBLE CONSTANT

The *Hubble constant* is given by the ratio of the expansion rate given in units of *km/sec* divided by the radius of the expansion in *Mpc*. The radius of expansion is equivalent to the radius of the light sphere with an origin at the time point when the Universe stopped contracting and started to expand.

$$H = \frac{\dot{\aleph}}{t.Mpc} = \frac{4\pi c\,X\,10^{-3}\sin\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\frac{km}{sec}}{t\,Mpc}, \tag{42}$$

for $t = 10^{10}\ light\ years = 3.069\,X\,10^3\,Mpc$, the Hubble constant, H_0, is 78.6 *km/sec·Mpc*. The experimental value is[18] $H_0=80 \pm 17$ *km/sec·Mpc*.

10. THE DENSITY OF THE UNIVERSE AS A FUNCTION OF TIME

The density of the Universe as a function of time $\rho_U(t)$ is given by the ratio of the

mass as a function of time and the volume as a function of time.

$$\rho_U(t) = \frac{m_U(t)}{V(t)} = \frac{m_U(t)}{\frac{4}{3}\pi\aleph(t)^3} = \frac{\frac{m_U}{2}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\right)}{\frac{4}{3}\pi\left(\left(\frac{2Gm_U}{c^2}+\frac{\frac{cm_U}{c^3}}{4\pi G}\right)-\frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\right)^3}, \tag{43}$$

for $t = 10^{10}$ *light years*, $\rho_U = 1.7\ X\ 10^{-32}\ g/cm^3$ The density of luminous matter of stars and gas of galaxies is about $\rho_U = 2\ X\ 10^{-31}\ g/cm^3$.[19-20]

11. THE POWER OF THE UNIVERSE AS A FUNCTION OF TIME, $P_U(t)$

From $E = mc^2$ and Eq. (34),

$$P_U(t) = \frac{c^5}{8\pi G}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right)\right) \tag{44}$$

For $t = 10^{10}$ *light years*, $P_U(t) = 2.88\ X\ 10^{51}\ W$. The observed power is consistent with that predicted.

12. THE TEMPERATURE OF THE UNIVERSE AS A FUNCTION OF TIME

The temperature of the Universe as a function of time, $T_U(t)$, follows from the Stefan-Boltzmann law.

$$T_U(t) = \left(\frac{1}{1+\frac{Gm_U(t)}{c^2\aleph(t)}}\right)\left[\frac{R_U(t)}{e\sigma}\right]^{\frac{1}{4}} = \left(\frac{1}{1+\frac{Gm_U(t)}{c^2\aleph(t)}}\right)\left[\frac{\frac{P_U(t)}{4\pi\aleph(t)^2}}{e\sigma}\right]^{\frac{1}{4}}. \tag{45}$$

The calculated uniform temperature is about 2.7 K which is in agreement with the observed microwave background temperature.[10]

13. POWER SPECTRUM OF THE COSMOS

The power spectrum of the cosmos, as measured by the Las Campanas Survey,

generally follows the prediction of cold dark matter on the scales of 200 million to 600 million light-years. However, the power increases dramatically on scales of 600 million to 900 million light-years. The infinitesimal temporal displacement, $d\tau^2$, is given by Eq. (13).

The relationship between the proper time and the coordinate time is

$$\tau = t\sqrt{1 - \frac{2Gm_U}{c^2 r}} = t\sqrt{1 - \frac{r_g}{r}}. \tag{46}$$

The power maximum in the proper frame occurs at

$$\tau = 5\ X\ 10^9\ \textit{light years}\sqrt{1 - \frac{3.12\ X\ 10^{11}\ \textit{light years}}{3.22\ X\ 10^{11}\ \textit{light years}}} = 880\ X\ 10^6\ \textit{light years}. \tag{47}$$

The power maximum of the current observable Universe is predicted to occur on the scale of 880 X 10^6 *light years*. There is excellent agreement between the predicted value and the experimental value of 600–900 X 10^6 *light years.*[17]

14. THE EXPANSION/CONTRACTION ACCELERATION, $\ddot{\aleph}$

The expansion/contraction acceleration rate, $\ddot{\aleph}$, is given by the time derivative of Eq. (41).

$$\ddot{\aleph} = 2\pi\frac{c^4}{Gm_U}\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}\sec}\right) \doteq \ddot{\aleph} = H_o = 78.7\cos\left(\frac{2\pi t}{3.01\ X\ 10^5\ Mpc}\right)\frac{km}{sec\cdot Mpc} \tag{48}$$

The differential in the radius of the Universe, $\Delta\aleph$, due to its acceleration is given by $\Delta\aleph = 1/2\ddot{\aleph}t^2$. The differential in expanded radius for the elapsed time of expansion, $t = 10^{10}$ *light years* corresponds to a decease in brightness of a supernovae standard candle of about an order of magnitude of that expected where the distance is taken as $\Delta\aleph$. This result based on the predicted rate of acceleration of the expansion is consistent with the experimental observation.[21–23]

Furthermore, the microwave background radiation image obtained by the Boomerang telescope[24] is consistent with a Universe of nearly flat geometry since the commencement of its expansion. The data is consistent with a large offset radius of the Universe with a fractional increase in size since the commencement of expansion about 10 billion years ago.

15. THE PERIODS OF SPACETIME EXPANSION/CONTRACTION AND PARTICLE DECAY/PRODUCTION FOR THE UNIVERSE ARE EQUAL

The period of the expansion/contraction cycle of the radius of the Universe, T, is given by Eq. (35). It follows from the Poynting power theorem with spherical radiation that the transition lifetimes are given by the ratio of energy and the power of the transition.

$$\tau = \frac{energy}{power} = \frac{[\hbar\omega]}{\left[\frac{2\pi c}{[(2l+1)!!]^2}\left(\frac{l+1}{l}\right)k^{2l+1}\left|Q_{lm}+Q'_{lm}\right|^2\right]}$$

$$= \frac{1}{2\pi}\left(\frac{h}{e^2}\right)\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{[(2l+1)!!]^2}{2\pi}\left(\frac{l}{l+1}\right)\left(\frac{l+3}{3}\right)^2\frac{1}{(kr_n)^{2l}\omega}. \tag{49}$$

Exponential decay applies to electromagnetic energy decay $h(t) = e^{-\frac{1}{T}t}u(t)$. The coordinate time is imaginary because energy transitions are spacelike due spacetime expansion from matter to energy conversion. For example, the mass of the electron (a fundamental particle) is given by

$$\frac{2\pi\lambda_C}{\sqrt{\frac{2Gm_e}{\lambda_C}}} = \frac{2\pi\lambda_C}{v_g} = i\alpha^{-1}\,sec, \tag{50}$$

where v_g is Newtonian gravitational velocity [Eq. (6)]. When the gravitational radius r_g is the radius of the Universe, the proper time is equal to the coordinate time by Eq. (15), and the gravitational escape velocity v_g of the Universe is the speed of light. Replacement of the coordinate time, t, by the spacelike time, it, gives

$$h(t) = Re\left[e^{-i\frac{1}{T}t}\right] = \cos\frac{2\pi}{T}t, \tag{51}$$

where the period is T [Eq. (35)]. The continuity conditions based on the constant maximum speed of light (Maxwell's equations) are given by Eqs. (16). The continuity conditions based on the constant maximum speed of light (Schwarzschild metric) are given by Eqs. (17–18). The periods of spacetime expansion/contraction and particle decay/production for the Universe are equal because only the particles which satisfy Maxwell's equations and the relationship between proper time and coordinate time imposed by the Schwarzschild metric may exist.

16. WAVE EQUATION

The general form of the light front wave equations is given by Eq. (1). The equation

of the radius of the Universe, $\aleph$, may be written as

$$\aleph = \left(\frac{2Gm_U}{c^2} + \frac{\frac{cm_U}{c^3}}{4\pi G} \right) - \frac{\frac{cm_U}{c^3}}{4\pi G} \cos\left(\frac{2\pi}{\frac{2\pi G m_U}{c^3} sec} \left(t - \frac{\aleph}{c} \right) \right) m, \tag{52}$$

which is a solution of the wave equation for a light wave front.

17. CONCLUSION

Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and special, and general relativity are unified.

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